FLOW OF A RAREFIED GAS IN A PLANE CHANNEL OF FINITE LENGTH FOR A WIDE RANGE OF KNUDSEN NUMBERS

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The flow of a rarefied gas in an infinite channel has been studied quite extensively (see, e.g., [1-3]). But in reality, channel lengths of finite length have been considered in [4-13].

The Clausing equation was solved in [4], but this equation is correct only in the free-molecular regime. In [5] a correction due to end effects was found for continuum flow. Using the results of [5] and taking into account slip, a formula was obtained in [6] for arbitrary Knudsen number. But in the free-molecular regime this formula disagrees significantly with the results given in [4]. In [7] an empirical formula was obtained for the flux of gas in a finite circular capillary for arbitrary Knudsen number and arbitrary channel length. Nearly free-molecular flow regimes were considered in [8, 9]. In [10] a solution was derived using the BGK equations and the moment method; here the distribution function is represented as a linear combination of four Maxwellian distributions. A simple analytical formula for the flux was obtained, which reproduced the numerical results found in [10] to within an error of 10% for the case when the ratio of channel width to height varied from 6 to 0.5, the Knudsen number varied from 5 to 0.5, and the ratio of pressures at the ends of the channel varied from 0.8 to 0.1. Unfortunately the numerical results calculated in [10] were not presented in the paper.

Flow in a plane and circular channel of finite length were considered in [11] and [12], respectively. The following simplifying assumptions were made: the density gradient along the channel was constant, and the velocity in the channel had only a longitudinal component depending only on the transverse coordinates. These assumptions are valid only for sufficiently long channels. The same assumptions were used in [13] in a study of the nonisothermal motion of a gas in a plane channel. Therefore the flow of a gas in a finite channel has been considered either for a restricted range of Knudsen numbers or using assumptions which are correct only for long channels.

In the present paper we consider the flow of a gas in a plane channel of finite length on the basis of the linearized BGK equation and the moment method. Unlike [11, 12], here no assumptions about the flow field are made, and hence we can take into account the variation of the velocity profile and the nonlinear dependence of the density on the longitudinal coordinate near the ends of the channel. The integral equations are solved numerically using the Krylov–Bogolyubov method. In the case when the channel length is much larger than the mean free path of a molecule, we can find a simple relation between the flux of gas in a finite channel and the flux in an infinite channel at the same Knudsen number.

1. We consider the steady flow of a gas between two parallel, infinitely wide plates forming the planes $y = \pm a$ and having length ℓ along the flow. The two vessels joining this channel contain the same kind of gas at the same temperature T, but at different densities n_1 and n_2, as shown in Fig. 1. Because of the density difference, the gas moves along the x-direction.

We introduce the scales a, n_1, $\beta = (2RT)^{1/2}$, $n_1\beta^{-3/2}$, $\eta_1 = n_1mv\lambda_1/2$ for length, density n, velocities $\mathbf{c}$ and $\mathbf{u}$, distribution function f, and viscosity η, respectively. Here R is the gas constant; m is the mass of a molecule; $v = (8RT/\pi)^{1/2}$ is the thermal velocity of the molecules; λ_1 is the mean free path of a molecule in the first vessel. All expressions below will be written in these units.

We assume that the density difference is much less than the average density $|\Delta n| = |n_2 - 1| \ll 1$, the reflection of molecules from the walls is diffuse, and that the molecules

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entering the channel from the two ends have a Maxwellian distribution function:

$$x = 0 \; c_x \geqslant 0, \; f = f_1 = \pi^{-3/2} \exp(-c^2),$$
$$x = L \; c_x \leqslant 0, \; f = f_2 = n_2 f_1 \; (L = l/a).$$

This means that the variation of the distribution function near the left end of the channel is not taken into account (as in [10-12]). Therefore our results will be valid only for sufficiently long (but finite) channels. The effect of the intake region on the flux of the gas will be estimated below. We note that the formulation of the problem given here correctly describes the transpiration of gas from one end of the channel to the other, which is a case of practical interest. It was shown in [10] that the deviation of the temperature of the gas in the channel from the equilibrium value, and the effect of this deviation on the flow parameters does not exceed 0.5%, and therefore the temperature can be assumed to be constant in the entire flow field.

The BGK equation for the distribution function is used as the starting point. It has the form

$$\mathbf{c}\,\partial f/\partial \mathbf{r} = \delta(f^0 - f), \tag{1.1}$$

where $\delta = \sqrt{\pi}a/2\lambda_1$ is the reciprocal of the Knudsen number;

$$f^0(\mathbf{r}, \mathbf{c}) = \frac{n(\mathbf{r})}{\pi^{3/2}} \exp[-(\mathbf{c} - \mathbf{u}(\mathbf{r}))^2];$$

$$n(\mathbf{r}) = \iiint_{-\infty}^{+\infty} f(\mathbf{r}, \mathbf{c})\, d\mathbf{c}; \tag{1.2}$$

$$\mathbf{u}(\mathbf{r}) = \iiint_{-\infty}^{+\infty} f(\mathbf{r}, \mathbf{c})\, \mathbf{c}\, d\mathbf{c}; \tag{1.3}$$

c is the velocity of a molecule; $\mathbf{r} = \mathbf{r}(x, y)$.

Because the density difference is small, the unknown distribution function can be represented as

$$f(\mathbf{r}, \mathbf{c}) = f_1[1 + h(\mathbf{r}, \mathbf{c})\Delta n]. \tag{1.4}$$

Then from the definitions (1.2) and (1.3)

$$n(\mathbf{r}) = 1 + q_1(\mathbf{r})\Delta n, \quad q_1(\mathbf{r}) = \iiint_{-\infty}^{+\infty} f_1 h(\mathbf{r}, \mathbf{c})\, d\mathbf{c},$$
$$u_x(\mathbf{r}) = q_2(\mathbf{r})\Delta n, \quad q_2(\mathbf{r}) = \iiint_{-\infty}^{+\infty} f_1 h(\mathbf{r}, \mathbf{c})\, c_x d\mathbf{c}, \tag{1.5}$$
$$u_y(\mathbf{r}) = q_3(\mathbf{r})\Delta n, \quad q_3(\mathbf{r}) = \iiint_{-\infty}^{+\infty} f_1 h(\mathbf{r}, \mathbf{c})\, c_y d\mathbf{c}.$$

Substituting (1.4) into (1.1), it is straightforward to obtain a linearized BGK equation, and integration of this equation along the characteristics gives an expression for the perturbation function

$$h(\mathbf{r}, \mathbf{c}) = \delta \int_0^{s_0} [q_1(\mathbf{r}) + 2c_x q_2(\mathbf{r}) + 2c_y q_3(\mathbf{r})] \exp\left(-\frac{\delta s}{c_p}\right)\frac{ds}{c_p} + h_0 \exp\left(-\frac{\delta s_0}{c_p}\right), \tag{1.6}$$

where $s = |\mathbf{r} - \mathbf{r}'|$; c_p is the projection of the velocity c onto the xy-plane; s_0 is the distance between the observation point and the boundary of the flow field in the direction c_p (see Fig. 1); h_0 is the perturbation function of the molecules emitted from the boundary with velocity c_p to the point of observation. According to the above assumptions and the condition that the gas cannot penetrate the walls of the channel, we have for h_0

$$h_0|_{y=\pm 1} = q_4(x) = -\frac{\iiint_{c_y \gtrless 0} f_1 h c_y d\mathbf{c}}{\iiint_{c_y \lessgtr 0} f_1 c_y d\mathbf{c}}. \tag{1.7}$$

The reduced number of molecules n_w incident per unit time on a unit area of the wall is related to q_4 by $n_w(x) = 1 + q_4(x)\Delta n$. Substituting (1.6) into (1.2), (1.3), and (1.7), we have a system of integral equations

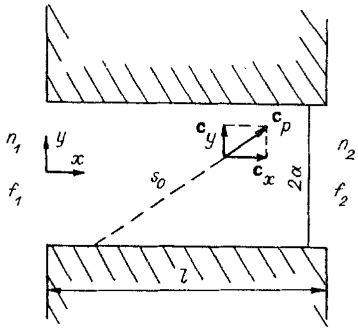

Fig. 1

$$q_i(x, y) = \int_0^L \int_{-1}^{1} \sum_{j=1}^{3} K_{ij}(x, y, x', y')\, q_j(x', y')\, dy' dx' + \int_0^L K_{i4}(x, y, x')\, q_4(x')\, dx' + \Phi_i(x, y), \quad 1 \leqslant i \leqslant 3, \tag{1.8}$$

$$q_4(x) = \int_0^L \int_{-1}^{1} \sum_{j=1}^{3} K_{4j}(x, x', y')\, q_j(x', y')\, dy' dx' + \int_0^L K_{44}(x, x')\, q_4(x')\, dx' + \Phi_4(x).$$

Here

$$K_{11} = \frac{\delta}{\pi} \frac{1}{s} I_0(\delta s); \quad K_{12} = \frac{\delta}{\pi} \frac{2}{s^2} I_1(\delta s)(x - x'); \quad K_{13} = \frac{\delta}{\pi} \frac{1}{s} I_1(\delta s)(y - y');$$

$$K_{14} = \frac{1}{\pi} \left[\frac{1}{s_1^2} I_1(\delta s_1)(1 + y) + \frac{1}{s_2^2} I_1(\delta s_2)(1 - y) \right]; \quad K_{21} = \frac{\delta}{\pi} \frac{1}{s^2} I_1(\delta s)(x - x');$$

$$K_{22} = \frac{\delta}{\pi} \frac{2}{s^3} I_2(\delta s)(x - x')^2; \quad K_{23} = \frac{\delta}{\pi} \frac{2}{s^3} I_2(\delta s)(x - x')(y - y');$$

$$K_{24} = \frac{1}{\pi} \left[\frac{1}{s_1^3} I_2(\delta s_1)(1 + y)(x - x') + \frac{1}{s_2^3} I_2(\delta s_2)(1 - y)(x - x') \right];$$

$$K_{31} = \frac{\delta}{\pi} \frac{1}{s^2} I_1(\delta s)(y - y'); \quad K_{32} = \frac{\delta}{\pi} \frac{2}{s^3} I_2(\delta s)(x - x')(y - y');$$

$$K_{33} = \frac{\delta}{\pi} \frac{2}{s^3} I_2(\delta s)(y - y')^2; \quad K_{34} = \frac{1}{\pi} \left[\frac{1}{s_1^3} I_2(\delta s_1)(1 + y)^2 - \frac{1}{s_2^3} I_2(\delta s_2)(1 - y)^2 \right];$$

$$K_{41} = \frac{\delta}{\sqrt{\pi}} \frac{2}{s_3^2} I_1(\delta s_3)(1 + y'); \quad K_{42} = \frac{\delta}{\sqrt{\pi}} \frac{4}{s_3^3} I_2(\delta s_3)(1 + y')(x - x');$$

$$K_{43} = -\frac{\delta}{\sqrt{\pi}} \frac{4}{s_3^3} I_2(\delta s_3)(1 + y')^2; \quad K_{44} = \frac{8}{\sqrt{\pi}} \frac{1}{s_4^3} I_2(\delta s_4);$$

$$\Phi_i = \frac{(-1)^i}{\delta} \int_{-1}^{1} K_{2i}(x, y, L, y')\, dy' \qquad (1 \leqslant i \leqslant 3);$$

$$\Phi_4 = -\frac{1}{\sqrt{\pi}} \int_{-1}^{1} \frac{1}{s_5^3} I_2(\delta s_5)(1 + y')(L - x)\, dy',$$

$$s = [(x - x')^2 + (y - y')^2]^{1/2}; \quad s_1 = [(x - x')^2 + (1 + y)^2]^{1/2};$$

$$s_2 = [(x - x')^2 + (1 - y)^2]^{1/2}; \quad s_3 = [(x - x')^2 + (1 + y')^2]^{1/2};$$

$$s_4 = [4 + (x - x')^2]^{1/2}; \quad s_5 = [(L - x)^2 + (1 + y')^2]^{1/2};$$

$$I_n(t) = \int_0^\infty c^n \exp\left(-c^2 - \frac{t}{c}\right) dc.$$

2. We chose the Krylov—Bogolyubov method [14] to solve this system of integral equations. The segment [0, L] is split up into k intervals $[x_{\ell-1}, x_\ell]$ ($\ell = 1, 2, \ldots, k$), where $x_0 = 0$, $x_k = L$, and the segment [−1, 1] is split up into n intervals $[y_{m-1}, y_m]$ ($m = 1, 2, \ldots, n$), where $y_0 = -1$ and $y_n = 1$. Then the system of integral equations (1.8) transforms into a system of algebraic equations

$$q_i^{lm} = K_{ij}^{lmps} q_j^{ps} + K_{i4}^{lmp} q_4^p + \Phi_i^{lm} \qquad (1 \leqslant i \leqslant 3), \tag{2.1}$$

$$q_4^l = K_{4j}^{lps} q_j^{ps} + K_{44}^{lp} q_4^p + \Phi_i^l.$$

Here on the right hand sides of these equations a summation is understood over repeating upper and lower indices, and we have adopted the notation:

$$q_i^{lm} \approx q_i(\tilde{x}_l, \tilde{y}_m),\ q_4^l \approx q_4(\tilde{x}_l),\ K_{ij}^{lmps} = \int\limits_{x_{p-1}}^{x_p}\int\limits_{y_{s-1}}^{y_s} K_{ij}(\tilde{x}_l, \tilde{y}_m, x', y')\,dy'dx',$$

$$K_{i4}^{lmp} = \int\limits_{x_{p-1}}^{x_p} K_{i4}(\tilde{x}_l, \tilde{y}_m, x')\,dx',\ K_{4j}^{lps} = \int\limits_{x_{p-1}}^{x_p}\int\limits_{y_{s-1}}^{y_s} K_{4j}(\tilde{x}_l, x', y')\,dy'dx',$$

$$K_{44}^{lp} = \int\limits_{x_{p-1}}^{x_p} K_{44}(\tilde{x}_l, x')\,dx',\ \Phi_i^{lm} = \Phi_i(\tilde{x}_l, \tilde{y}_m),\ \Phi_4^l = \Phi_4(\tilde{x}_l),$$

$$x_{l-1} < \tilde{x}_l < x_l,\ y_{m-1} < \tilde{y}_m < y_m.$$

In view of the symmetry of the problem and the fact that it is linear, the functions q_i have the property

$$\begin{gathered} q_1(x, y) = q_1(x, -y) = 1 - q_1(L - x, y), \\ q_2(x, y) = q_2(x, -y) = q_2(L - x, y), \\ q_3(x, y) = -q_3(x, -y) = q_3(L - x, y),\ q_4(x) = 1 - q_4(L - x), \end{gathered} \tag{2.2}$$

and use of this property allows one to reduce the order of the system of algebraic equations (2.1) by a factor of four.

The system (2.1) was solved by the Gauss–Seidel iteration method. It can be shown [15] that the necessary and sufficient conditions for the convergence of the scheme are satisfied for the range of δ considered here. The calculations were done numerically on a computer.

3. In the case when the channel length is much greater than the mean free path of a molecule ($\delta L \gg 1$), the flow field near the ends is similar for fixed δ and different L. This similarity means that we can eliminate the calculation of cases with large values of δ_L. To prove the similarity, we consider the flow of a gas in a semi-infinite channel whose end lies in the cross section x = 0, where the flow is due to a small constant density gradient $\nu = \partial n_\infty/\partial x$ in the limit $x \to \infty$. Obviously at infinity ($x \to \infty$), the flow field will correspond to flow in an infinite channel with the same density graident ν.

The boundary condition at x = 0 is taken to be the same condition as that for the left end of the finite channel. The problem can then be linearized in the parameter ν, i.e., the distribution function f_∞ can be represented in the form $f_\infty = f_1[1 + h_\infty(\mathbf{r}, \mathbf{c})\nu]$. The moments of the distribution function are written as

$$\begin{gathered} n_\infty(\mathbf{r}) = 1 + p_1(\mathbf{r})\nu,\ u_{x\infty}(\mathbf{r}) = p_2(\mathbf{r})\nu, \\ n_{w\infty}(x) = 1 + p_4(x)\nu,\ u_{y\infty}(\mathbf{r}) = p_3(\mathbf{r})\nu. \end{gathered} \tag{3.1}$$

Here the functions $p_i(\mathbf{r})$ ($1 \le i \le 4$) are determined in exactly the same way as the functions $q_i(\mathbf{r})$. The corresponding moments of the distribution function in the left half of the finite channel (for the same value of δ) will approach (3.1) as the length of the channel increases if

$$\nu(\delta, L) = \left.\frac{\partial n}{\partial x}\right|_{x=L/2} = \left.\frac{\partial q_1}{\partial x}\right|_{x=L/2}\Delta n. \tag{3.2}$$

In other words, there always exists a channel length, such that the following relation is satisfied for the left half of the channel (to within the approximations assumed here and subject to the condition (3.2))

$$q_i(\mathbf{r}, \delta, L)\Delta n = p_i(\mathbf{r}, \delta)\nu(\delta, L),\ 1 \leqslant i \leqslant 4. \tag{3.3}$$

Suppose we know the flow field $q_i(\mathbf{r}, \delta, L^*)$ for a channel length L^* for which the flow in its central part corresponds to flow in an infinite channel with the density gradient $\nu(\delta, L^*)$ (to within the accuracy of the calculation). Then using (3.3), the flow field in a channel of arbitrary length $L > L^*$ in the region $0 \le x \le L^*/2$ can be expressed in terms of $q_i(\mathbf{r}, \delta, L^*)$ as follows:

$$q_i(\mathbf{r}, \delta, L) = \frac{\nu(\delta, L)}{\nu(\delta, L^*)} q_i(\mathbf{r}, \delta, L^*), \qquad 1 \leqslant i \leqslant 4. \tag{3.4}$$

In the region $L^*/2 \le x \le L/2$, the flow field corresponds to an infinite channel, and therefore the functions q_i are given by

$$q_1(\mathbf{r}, \delta, L) = q_4(x, \delta, L) = \frac{\nu(\delta, L)}{\Delta n}\left(x - \frac{L}{2}\right) + \frac{1}{2}, \qquad (3.5)$$

$$q_2(\mathbf{r}, \delta, L) = q_2\left(x = \frac{L^*}{2}, y, \delta, L\right), \quad q_3(\mathbf{r}, \delta, L) = 0.$$

We note that in view of (3.2) the ratio $\nu(\delta, L)/\Delta n$ is independent of Δn. The flow field for $L/2 \le x \le L$ is calculated from (2.2).

To find a relation between $\nu(\delta, L)$ and $\nu(\delta, L^*)$, we use the fact that both of the relations (3.4) and (3.5) are correct for the cross section $x = L^*/2$. Equating the right-hand sides of these relations for q_1 and using the fact that $q_1(x = L^*/2, y, \delta, L^*) = 1/2$, it is not difficult to obtain

$$\frac{\Delta n}{\nu(\delta, L)} - L = \frac{\Delta n}{\nu(\delta, L^*)} - L^*. \qquad (3.6)$$

The quantity of most interest in practice is the flux of gas through a cross section of the channel. It can be determined using the formula

$$Q = \frac{L}{2\Delta n}\int_{-1}^{1} u_x(x, y)\,dy = \frac{L}{2}\int_{-1}^{1} q_2(x, y)\,dy. \qquad (3.7)$$

Substituting (3.4) for q_2 into (3.7) and using (3.6), we have

$$Q = \frac{L}{L + \Delta L} Q_\infty; \qquad (3.8)$$

$$Q_\infty = \frac{\Delta n}{2\nu(\delta, L^*)}\int_{-1}^{1} q_2(x, y, \delta, L^*)\,dy; \qquad (3.9)$$

$$\Delta L = \frac{\Delta n}{\nu(\delta, L^*)} - L^*. \qquad (3.10)$$

Since the flow field in the central part of a channel of length L^* corresponds to flow in an infinite channel with the density gradient $\nu(\delta, L^*)$, Q_∞ by definition [3] corresponds to the reduced flux in an infinite channel. The quantities Q_∞ and ΔL do not depend on the channel length and can be determined by calculating the flow field for L^* and using the equations (3.2), (3.9), and (3.10).

Therefore the flow field calculated for L^* using (3.4) through (3.6) and (3.8) can be used to find the flow field and flux of gas in a channel of any length $L > L^*$. The calculations show that, to within an error less than 2%, the necessary condition for the applicability of these formulas is $\delta L \ge 40$.

4. In Fig. 2 the solid curves give the dependence of the gas flux Q on the reciprocal of the Knudsen number. The error in the calculations is less than 2%. The accuracy of the calculation is determined by comparing the values of the flux for different numbers of grid points in both coordinates x and y. In every case the variation of the flux in different cross sections of the channel is within the accuracy of the calculation. In the free-molecular regime ($\delta = 0$), the results of the calculation agree with the data of [4], since in this case the four equations of (1.8) reduce to the Clausing equation.

For comparison, the results of [11] are presented in Fig. 2 (dashed curves). Large discrepancies are observed over the entire range of Knudsen numbers. The authors of [11] themselves noted the deviation from the exact solution in the free-molecular regime. The discrepancy in the intermediate and viscous regimes can be explained by comparing the density fields.

It was assumed in [11] that the density is constant in each cross section of the channel and varies linearly from n_1 to n_2. In the present paper the density was calculated. Figure 3 shows the dependence of the function q_1 (related to the density by the first equation of (1.5)) on the longitudinal coordinate. Curves 1 and 2 correspond to $L = 4$ and 10, $\delta = 10$ and 4. The solid curves correspond to the density near the channel wall, the dashed curves correspond to the center of the channel, and the dash-dotted curve to the flow field assumed in [11] for all channel lengths and Knudsen numbers.

TABLE 1

δ	L^*	ΔL	Q_∞	Q'_∞ [3]
10	4	18,8	4,37	4,40
4	10	10,1	2,41	2,4472
2	20	7,47	1,83	1,8440
1	40	6,46	1,59	1,5942
0,4	100	7,14	1,54	1,5482
0,2	200	10,6	1,63	1,6408
0,1	400	12,6	1,80	1,8075

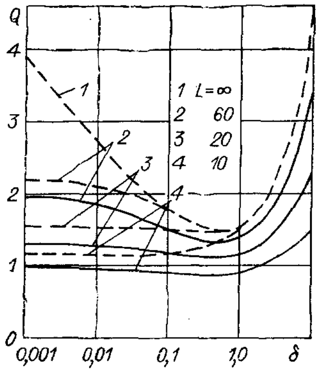

Fig. 2

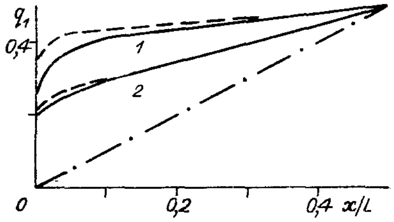

Fig. 3

We see from Fig. 3 that in the left portion of the channel the gas density is not constant over a cross section of the channel and varies nonlinearly along the channel. The density at the end of the channel (x = 0) is not equal to n_1. This is explained by the fact that at x = 0, a Maxwellian distribution function f_1 was specified for the right half of velocity space. The distribution function for the left half of velocity space is found from the solution of the kinetic equation and in general is not the same as f_1. When moments of the distribution function are calculated at the end of the channel over the entire velocity space, a density is obtained which is different from n_1.

In the central portion of the channel, the density gradient is constant but it is significantly smaller than the density field assumed in [11]. For example, with δ = 4, L = 10 (curve 2) the ratio of these two gradients is approximately equal to two. The fluxes also differ by about a factor of two in this case. Hence the density field assumed in [11] leads to large errors over the entire range of Knudsen numbers.

The gas flux for $\delta L \geq 40$ was calculated using (3.8). Table 1 gives the values of ΔL and Q_∞ for several values of δ. In the second column we give the minimum length for which (3.8) is valid, within the accuracy limits of the calculation. For comparison, we give in the fifth column the reduced flux in an infinite channel Q_∞ obtained in [3] upon conversion of the number δ into our length scale. For δ = 10 we compared our results with [13], since calculations were done in [3] only up to δ = 5. We see that the differences from the results of [3] are within the computational error.

We note that ΔL is comparable to and may exceed L^* for large values of δ. This means that the flux calculated from (3.8) may be several times smaller than the flux in an infinite channel. The data in Table 1 can be used to estimate the channel length for which the error due to end effects is smaller than a given value. Estimates of this kind are essential in experimental design.

5. One would expect that the effect of the intake region on the gas flux would be a maximum for the viscous flow regime. To estimate this effect the total resistance of the channel L/G_t is written as a sum (as done in [6])

$$L/G_t = L/G + 1/G_0, \quad (5.1)$$

where G_t is the reduced flux in a channel with the effect of the intake region taken into account; L/G is the resistance of the inner part of the channel; $1/G_0$ is the resistance of the intake region. Using this relation, it is simple to obtain an expression for the relative difference between G_t and G:

$$\gamma = (G - G_t)/G_t \cdot 100\% = G/G_0 L \cdot 100\%. \quad (5.2)$$

The value $1/G_0$ is comparable to the resistance of an infinitely thin slot; the flux through a channel of this type was found in [16] for the case of a continuous medium and is given by $G_0 = \delta\pi/16$. Substituting the value of G from the present paper and the value of G_0 from [16] into (5.2), we obtain γ = 8, 6, and 3% for the channel lengths L = 10, 20, and 60, respectively, at δ = 10. The effect of the intake region becomes less significant as the number δ decreases.

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